

List of Integral / Space Transforms

Laplace Transforms

$$F(s) = \int_{-\infty}^{\infty} f(x)e^{-st} dt$$

$f(t)$	$F(s)$
1	$\frac{1}{s}$
$t^n, (n=0,1,2,\dots)$	$\frac{n!}{s^{n+1}}$
$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{at}	$\frac{1}{s-a}$
$e^{at} f(t)$	$F(s-a)$
$t e^{at}$	$\frac{1}{(s-a)^2}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$\frac{e^{at} - e^{bt}}{a-b}$	$\frac{1}{(s-a)(s-b)}$
$\frac{ae^{at} - e^{bt}}{a-b}$	$\frac{s}{(s-a)(s-b)}$
$\sin(kt)$	$\frac{k}{s^2+k^2}$
$\cos(kt)$	$\frac{s}{s^2+k^2}$
$t \sin(kt)$	$\frac{2ks}{(s^2+k^2)^2}$
$t \cos(kt)$	$\frac{s^2-k^2}{(s^2+k^2)^2}$
$e^{at} \sin(kt)$	$\frac{k}{(s-a)^2+k^2}$
$e^{at} \cos(kt)$	$\frac{s-a}{(s-a)^2+k^2}$
$\frac{\sin at}{t}$	$\text{atan}\left(\frac{a}{s}\right)$
$\dot{f}(t)$	$sF(s) - f(0)$
$\int_0^t f(t) dt$	$\frac{1}{s} F(s)$
$f(t) * g(t)$	$F(s)G(s)$
$\delta(t)$	1
$\delta(t-t_0)$	e^{-st_0}
$u(t-a)$	$\frac{e^{-as}}{s}$
$u(t-a)f(t-a)$	$e^{-as}F(s)$

Z Transforms

$$F[z] = \sum_{n=-\infty}^{\infty} f[n]z^{-n}$$

- Geometric Series:

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$$

For most transforms, taking the inversion of the coefficients and arguments leads to the same transform, but the ROC is inverted. e.g.

$$\mathcal{Z}\{u[n]\} = \frac{z}{z-1} \quad \text{ROC } |z| > 1 \quad \mathcal{Z}\{-u[-n-1]\} = \frac{z}{z-1} \quad \text{ROC } |z| < 1$$

$f[n]$	$F[z]$	ROC
$x[n]$	$X[z]$	$r_2 < z < r_1$
$x[n-k]$	$z^{-k}X[z]$	$ z \neq 0$
$x[n+k]$	$z^kX[z]$	$ z \neq \infty$
a^{nx}	$X\left[\frac{z}{a}\right]$	$ a r_2 < z < a r_1$
$\delta[n]$	1	$\text{All } z$
$u[n]$	$\frac{z}{z-1} = \frac{1}{1-z^{-1}}$	$ z > 1$
$a^n u[n]$	$\frac{z}{z-a} = \frac{1}{1-az^{-1}}$	$ z > a $
$nu[n]$	$\frac{z}{(z-1)^2} = \frac{z^{-1}}{(1-z^{-1})^2}$	$ z > 1$
$n^2 u[n]$	$\frac{z(z+1)}{(z-1)^3} = \frac{z(1+z^{-1})}{(1-z^{-1})^2}$	$ z > 1$
$na^n u[n]$	$\frac{za}{(z-a)^2} = \frac{az^{-1}}{(1-az^{-1})^2}$	$ z > a $
$\cos(\omega_0 n) u[n]$	$\frac{z(z-\cos(\omega_0))}{z^2-2z\cos(\omega_0)+1} = \frac{1-z^{-1}\cos(\omega_0)}{1-2z^{-1}\cos(\omega_0)+z^{-2}}$	$ z > 1$
$\sin(\omega_0 n) u[n]$	$\frac{z\sin(\omega_0)}{z^2-2z\cos(\omega_0)+1} = \frac{z^{-1}\sin(\omega_0)}{1-2z^{-1}\cos(\omega_0)+z^{-2}}$	$ z > 1$
$a^n \cos(\omega_0 n) u[n]$	$\frac{z(z-a\cos(\omega_0))}{z^2-2az\cos(\omega_0)+1} = \frac{1-az^{-1}\cos(\omega_0)}{1-2az^{-1}\cos(\omega_0)+a^2z^{-2}}$	$ z > 1$
$a^n \sin(\omega_0 n) u[n]$	$\frac{az\sin(\omega_0)}{z^2-2az\cos(\omega_0)+z^2} = \frac{az^{-1}\sin(\omega_0)}{1-2az^{-1}\cos(\omega_0)+a^2z^{-2}}$	$ z > 1$
$\text{ramp}[n]$	$\frac{z}{(z-1)^2}$	$ z > 1$
e^{at}	$\frac{z}{z-e^{at}}$	$ z > 1$

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