List of Integral / Space Transforms

Laplace Transforms

$$F(s) = \int_{-\infty}^{\infty}\,f(x)e^{st}\,dt$$

$f(t)$	**$F(s)$**
$$1$$	$$\frac{1}{s}$$
$$t^n\,,(n=0,1,2,...)$$	$$\frac{n!}{s^{n+1}}$$
$$t^nf(t)$$	$$(-1)\stackrel{n}{F}(s)$$
$$e^{at}$$	$$\frac{1}{s-a}$$
$$e^{at}f(t)$$	$$F(s-a)$$
$$te^{at}$$	$$\frac{1}{(s-a)^2}$$
$$t^ne^{at}$$	$$\frac{n!}{(s-a)^{n+1}}$$
$$\frac{e^{at}-e^{bt}}{a-b}$$	$$\frac{1}{(s-a)(s-b)}$$
$$\frac{ae^{at}-e^{bt}}{a-b}$$	$$\frac{s}{(s-a)(s-b)}$$
$$\sin(kt)$$	$$\frac{k}{s^2+k^2}$$
$$\cos(kt)$$	$$\frac{s}{s^2+k^2}$$
$$t\sin(kt)$$	$$\frac{2ks}{(s^2+k^2)^2}$$
$$t\cos(kt)$$	$$\frac{s^2-k^2}{(s^2+k^2)^2}$$
$$e^{at}\sin(kt)$$	$$\frac{k}{(s-a)^2-k^2}$$
$$e^{at}\cos(kt)$$	$$\frac{s-a}{(s-a)^2+k^2}$$
$$\frac{\sin{at}}{t}$$	$$\text{atan}\left(\frac{a}{s}\right)$$
$$\dot{f}(t)$$	$$sF(s) -f(0)$$
$$\int_0^t{\,f(t)\,dt}$$	$$\frac{1}{s}\,F(s)$$
$$f(t) * g(t)$$	$$F(s)G(s)$$
$$\delta(t)$$	$$1$$
$$\delta(t-t_0)$$	$$e^{-st_0}$$
$$u(t-a)$$	$$\frac{e^{-as}}{s}$$
$$u(t-a)f(t-a)$$	$$e^{a-s}F(s)$$

Z Transforms

$$F[z] = \sum_{n=-\infty}^{\infty}{f[n]z^{-n}}$$

- Geometric Series:

$$\sum_{n=0}^{\infty}{a^n} = \frac{1}{1-a}$$

For most transforms, taking the inversion of the coefficients and arguments leads to the same transform, but the ROC is inverted. e.g.

$$\mathcal{Z}\{u[n]\}=\frac{z}{z-1}\quad ROC\,|z|>1$$ $$\mathcal{Z}\{-u[-n-1]\}=\frac{z}{z-1}\quad ROC\,|z|<1$$

$$f[n]$$	**$$F[z]$$**	**$$ROC$$**
$$x[n]$$	$$X[z]$$	$$r_2 < \|z\| < r_1$$
$$x[n-k]$$	$$z^{-k}X[z]$$	$$\|z\|\neq 0$$
$$x[n+k]$$	$$z^{k}X[z]$$	$$\|z\|\neq \infty$$
$$a^nx[n]$$	$$X\left[\frac{z}{a}\right]$$	$$\|a\|r_2<\|z\|<\|a\|r_1$$
$$\delta [n]$$	$$1$$	$$\text{All } z$$
$$u[n]$$	$$\frac{z}{z-1}=\frac{1}{1-z^{-1}}$$	$$\|z\| > 1$$
$$a^nu[n]$$	$$\frac{z}{z-a}=\frac{1}{1-az^{-1}}$$	$$\|z\| > \|a\|$$
$$nu[n]$$[1]	$$\frac{z}{(z-1)^2}=\frac{z^{-1}}{(1-z^{-1})^2}$$	$$\|z\| > 1$$
$$n^2u[n]$$	$$\frac{z(z+1)}{(z-1)^3}=\frac{1+z^{-1}}{z(1-z^{-1})^2}$$	$$\|z\| > 1$$
$$na^nu[n]$$	$$\frac{za}{(z-a)^2}=\frac{az^{-1}}{(1-az^{-1})^2}$$	$$\|z\| > \|a\|$$
$$\cos(\omega_0 n)u[n]$$	$$\frac{z(z-\cos(\omega_0))}{z^2-2z\cos(\omega_0)+1}=\frac{1-z^1\cos(\omega_0)}{1-2z^{-1}\cos(\omega_0)+z^{-2}}$$	$$\|z\| > 1$$
$$\sin(\omega_0 n)u[n]$$	$$\frac{z\sin(\omega_0)}{z^2-2z\cos(\omega_0)+1}=\frac{z^{-1}\sin(\omega_0)}{1-2z^{-1}\cos(\omega_0)+z^{-2}}$$	$$\|z\| > 1$$
$$a^n\cos(\omega_0 n)u[n]$$	$$\frac{z(z-a\cos(\omega_0))}{a^2-2az\cos(\omega_0)+1}=\frac{1-az^1\cos(\omega_0)}{1-2az^{-1}\cos(\omega_0)+a^2z^{-2}}$$	$$\|z\| > 1$$
$$a^n\sin(\omega_0 n)u[n]$$	$$\frac{az\sin(\omega_0)}{a^2-2az\cos(\omega_0)+z^2}=\frac{az^{-1}\sin(\omega_0)}{1-2az^{-1}\cos(\omega_0)+a^2z^{-2}}$$	$$\|z\| > 1$$
$$e^{at}$$	$$\frac{z}{z-e^{at}}$$	$$\|z\| > 1$$

1)

$ramp(t)$